2023

Penrith High School

HIGHER SCHOOL CERTIFICATE

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- In Questions 11–32 show relevant mathematical reasoning and/or calculations.
- Write your NESA ID below, on the Multiple-Choice Answer Sheet and the front of Booklets 1, 2 and 3.

Total marks: 100

Section I – 10 marks (pages 3-8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 9-30)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section.

	Differentiation & applications of Differential Calculus	Statistics	Algebra & Graphing Functions	Integration	Logarithms & Exponentials	Probability	Series & Financial mathematics	Trigonometry
Multiple Choice	/1 10	/1 9	/1 7	/1 1	/1 3	/2 4 6	/2 2 5	/1 8
							Total:	/10

Differentiation And applications of Differential Calculus	Statistics	Algebra & Graphing Functions	Integration	Logarithms & Exponentials	Probability	Series & Financial mathematics	Trigonometric functions
/21	/5	/6	/16	/6	/11	/13	/12
						Total:	/90

Student NESA Number _____

Teacher_____

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section.
Use the multiple-choice answer sheet for Questions 1-10.

---

1. The value of the definite integral $\int_{-4}^{4} x^3\,dx$ is given by

   A. 128

   B. -128

   C. 0

   D. 64

2. A snail starts moving in a garden and covers 6 metres in the first hour. If the snail covers only half the distance covered in the previous hour in each successive hour, how far will the snail be able to travel?

   A. 10.5 metres

   B. 11.25 metres

   C. 11.625 metres

   D. 12 metres

3. Which of the following expressions is equivalent to $4 + \ln x$?

   A. $\ln(4x)$

   B. $\ln(4e + x)$

   C. $\ln(e^4 x)$

   D. $\mathrm{e}^4 \ln(x)$

4. The discrete random variable X has the following distribution:

X	0	1	2	3
$P(X = x)$	0.15	a	0.35	$2b$

Given that $\mathrm{E(X)} = 1.5$, find the values of a and b.

A. $a = 0.15$ and $b = 0.7$

B. $a = 0.35$ and $b = 0.075$

C. $a = 0.35$ and $b = 0.15$

D. $a = 0.15$ and $b = 0.35$

5. \$1350 is invested into an account earning 3% p.a. for 5 years. Interest is compounded quarterly. Which expression correctly evaluates the amount of interest earned?

A. 1350×1.0075^{20}

B. 1350×1.03^{5}

C. $1350 \times 1.0075^{20} - 1350$

D. $1350 \times 1.03^{5} - 1350$

6. A dice is rolled twice and the uppermost numbers are added together.

Which of the following events has a probability of $\frac{1}{18}$?

A. Both numbers rolled are the same

B. The sum is 5

C. The sum is 3

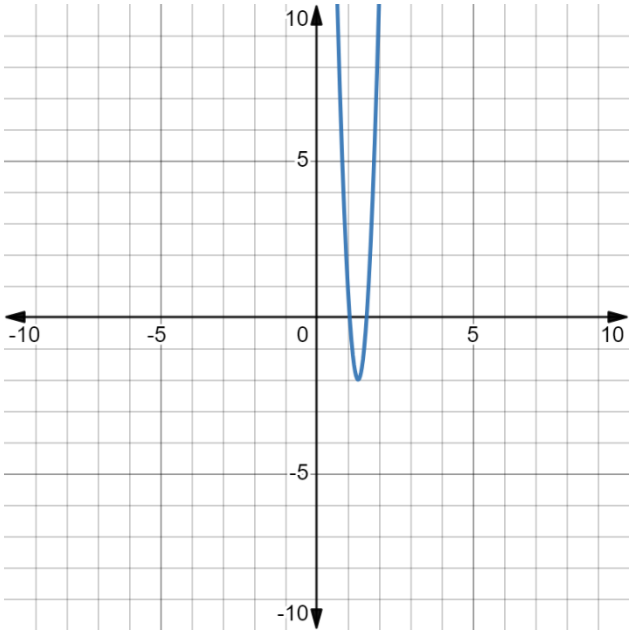
D. The sum is 12

7. The graph of $y = x^2$ undergoes the following sequence of transformations:

- *Translation 4 units right*
- *Dilation horizontally by a factor of* $\frac{1}{3}$
- *Translation 2 units down*
- *Dilation vertically by a factor of 3*

The graph of the resulting equation is given by

A.



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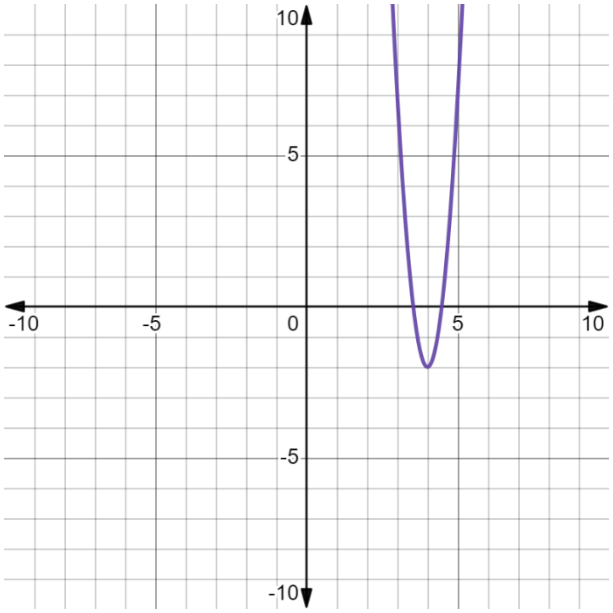

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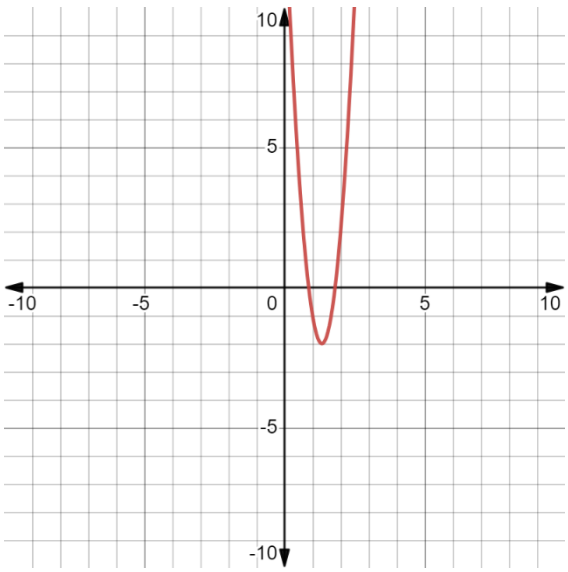

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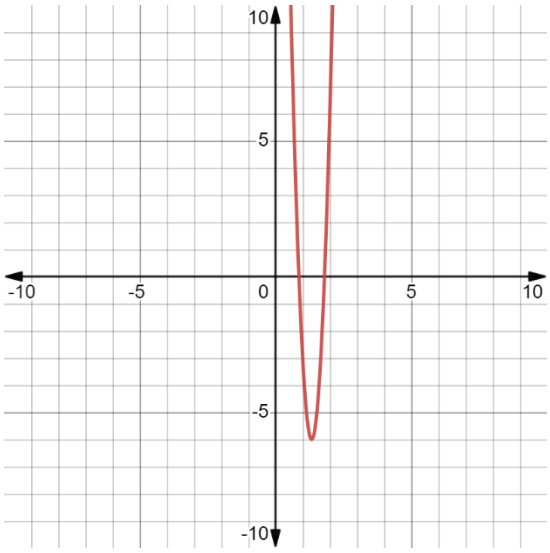
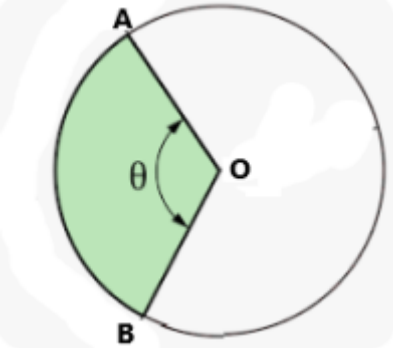


8. In a circle, an arc AB subtends an angle of θ radians at the centre shown below. The radius of the circle is $r\ cm$ and the perimeter of the minor sector AOB is $4\ cm$.



The angle θ and the area of the sector AOB is given by

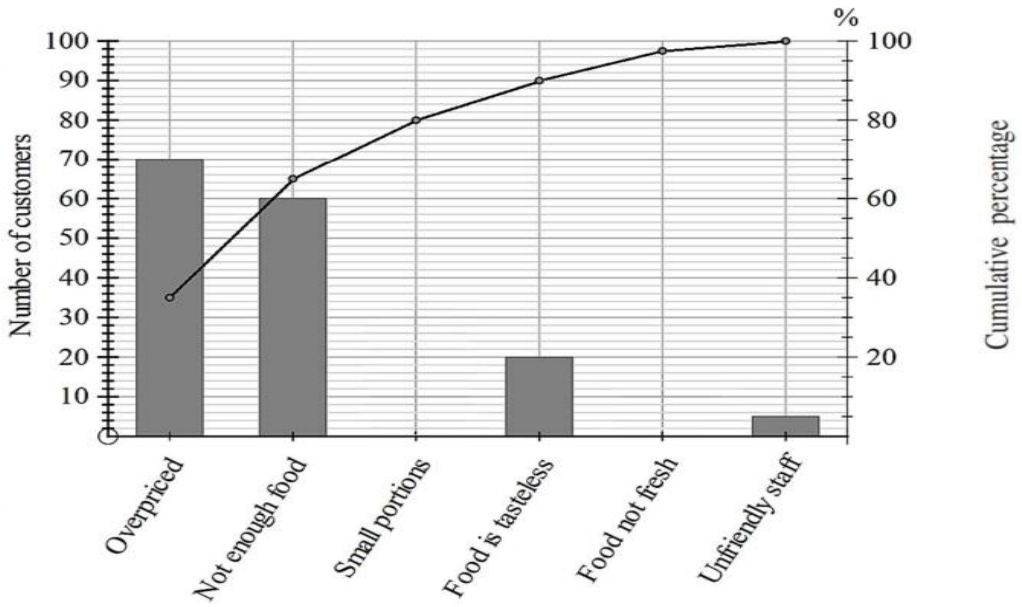
A. $\theta = \frac{4}{r} - 2$ and $A = 2r - r^2$

B. $\theta = \frac{4}{r}$ and $A = 2r$

C. $\theta = \frac{4}{r} - 2$ and $A = 4r - r^2$

D. $\theta = \frac{4}{r}$ and $A = 4r$

9.



The Pareto chart given above shows the complaints of 200 customers from a restaurant in the area. The columns for the "small portions" and "food not fresh" complaints are missing.

The approximate total number of complaints for "Small portions" is

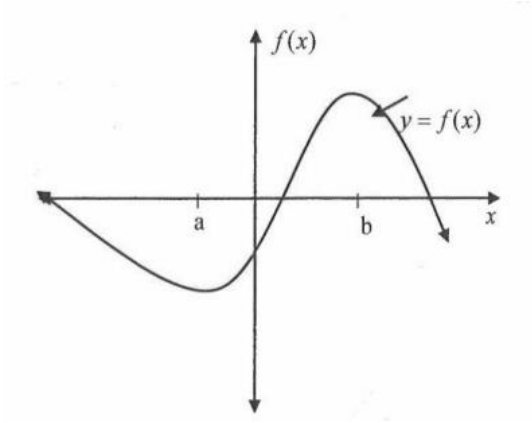
A. 45 people

B. 30 people

C. 40 people

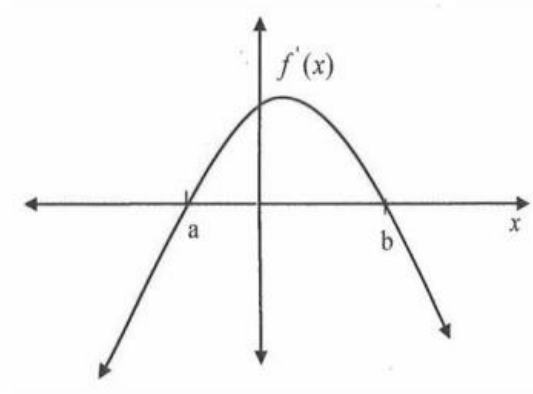
D. 80 people

10. The diagram given below shows the graph of $y = f(x)$.



Which of the following shows the graph of $y = f'(x)$?

A.



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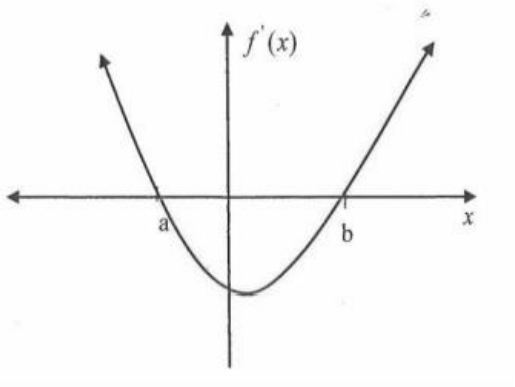

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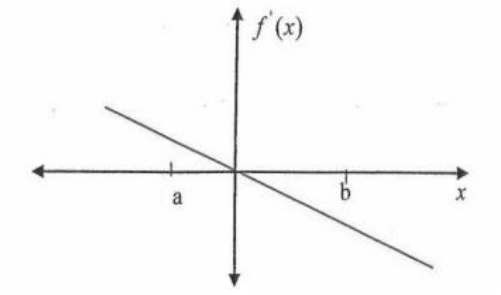

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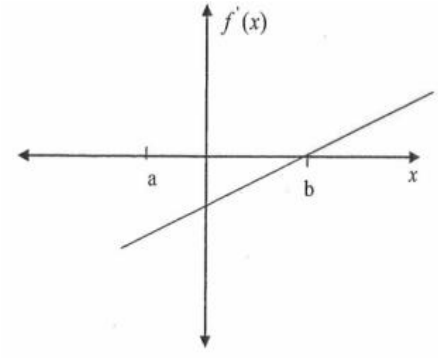

End of SECTION I

NESA Number: | | | | | | | | | |

2023

Penrith High School
HSC TRIAL
EXAMINATION

Teacher: _____ ____

Mathematics Advanced

Section II Answer Booklet 1

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 20 (30 marks)
Booklet 2 – Attempt Questions 21 – 26 (30 marks)
Booklet 3 – Attempt Questions 27 – 32 (30 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above.

Trigonometric Functions	Algebra & Graphing	Integration	Series & Financial Mathematics	Probability	Differentiation & Applications of Differentiation	Logarithms & Exponentials
/3 13	/6 14 17	/3 15	/3 16	/9 12 18	/3 11	/3 19

Please Turn Over

Question 11 (3 marks)

A student was asked to differentiate $f(x) = 2x^2 + x$ using the first Principles of Differential Calculus. The student began the solution as shown below.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Complete the solution. 3

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Question 12 (1 mark)

A and B are events of a sample space. Given that $P(A \cap B) = 0.15$ and $P(A|B) = 0.2$, calculate $P(B)$ for this sample space. 1

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Question 13 (3 marks)

Solve $2\sin 2\theta - 1 = 0$ for $0 \leq \theta \leq 2\pi$. 3

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Question 14 (3 marks)

Let $f(x) = x - 2$ and $g(x) = \frac{1}{x} + 1$,

a. find $g\big(f(x)\big)$. 1

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b. Find the domain and range of $g\big(f(x)\big)$ in interval notation. 2

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Question 15 (3 marks)

A function $f(x)$, has a derivative given by $f'(x) = 3x^2 - x + 6$. The function curve passes through the point $(0, 1)$. Find the equation of the function. 3

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Question 16 (3 marks)

The second term of an arithmetic sequence is 12. The fifteenth term is 64.

Calculate the 25th term. 3

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Question 17 (3 marks)

Consider the curve given by $f(x) = x^2 \ln|x|$ 3

Is $f(x) = x^2 \ln|x|$ an odd or an even function or neither? Justify your response.

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Question 18 (8 marks)

The probability distribution for a discrete random variable X is given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3 \\ k(6 - x) & x = 4, 5, 6 \end{cases}$$

a. Show that $k = \frac{1}{9}$. 1

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b. Complete the Probability distribution table given below. 2

X	1	2	3	4	5	6
$P(X = x)$						

c. Hence, calculate its expected value. 2

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d. Calculate the variance and the standard deviation correct to 2 decimal places. 2

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e. Find $P(2 \leq X \leq 4)$. 1

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Question 19 (3 marks)

Using the logarithm laws, show that $\frac{d}{dx}\left(\ln\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{1-x^2}$ 3

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Proceed to Answer Booklet 2 for Questions 20-24

Question 20 (8 marks)

Consider the curve given by $f(x) = 2x^3 - 5x^2 + 6$.

a. Find the coordinates of the stationary points and determine for their nature. 3

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b. Find the coordinates of any point(s) of inflection. 2

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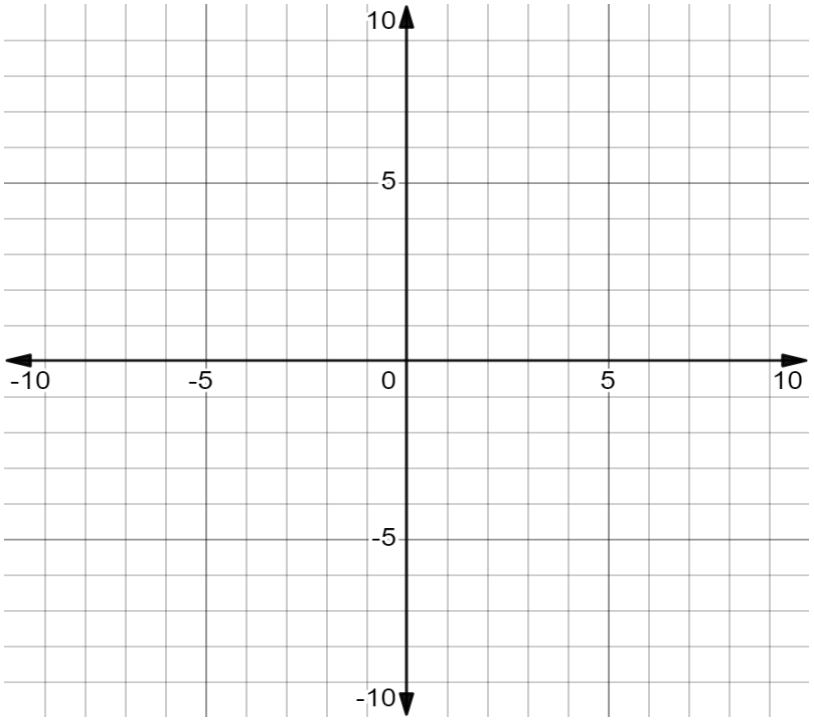
c. Hence, sketch the curve labelling the stationary points, point(s) of inflection (if any) and $y-$ intercept. Do Not determine the $x-$ intercepts of the curve. 3



Question 21 (2 marks)

Consider the series given by $1+3x+9x^2+27x^3+\cdots\ldots$

a. Show that it is a geometric series. 1

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b. Find the range of values of x for which the series has a limiting sum. 1

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Question 22 (6 marks)

a. Using the quotient rule, show that $\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = -cosec^2 x$ 3

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b. Evaluate $\int_5^8 \frac{x-2}{x^2-4x}\,dx$. Leave your answer in exact form. 3

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Question 23 (5 marks)

Six Penrith High School Year 12 Students participated in the javelin and discus throwing events at the Athletics Carnival this year. Their distances (in metres) were recorded in the table below.

Student Names	Distance (Javelin)	Distance (Discus)
Saatvik	8	18
Rithwik	60	81
Yesh	51	57
Angelo	48	48
Darwin	29	25
Lisa	36	47

The relationship between the two events is being examined. The distance thrown in javelin is the independent variable and the distance thrown in discus is the dependent variable.

a. Calculte the Pearson correlation coefficient, r, for the above data set correct to 3 decimal places. Explain clearly what this value indicates in this context. 2

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b. Calculate the equation of the least squres regression line, rounding numbers correct to 1 decimal place where necessary. 2

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c. Saatvik concluded after the event that "Yesh was able to throw his javelin a greater distance because he threw his discus a greater distance, and being successful in one skill transferred to the other skill". Saatvik's statement may not be correct. Why? 1

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Question 24 (5 marks)

The acceleration of a particle at any time t is given by $\ddot{x} = 2\cos\frac{\pi t}{3}$.

a. Find the velocity and displacement of the particle at any time t , given that the particle is initially at rest at the origin. 3

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b. Find the time when the particle first returns to the origin. 2

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Question 25 (2 marks)

A survey of a certain district showed that 4% of the families have 1 child, 34% have 2 children, 40% have 3 children, and 15% have more than 3 children. A family from the district is selected at random. Find the probability that the family will have at most 1 child. 2

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Question 26 (2 marks)

Show that the graph given by $f(x) = \frac{1}{1+e^{-x}}$, $x \geq 0$ is an increasing function for all values of x in the given domain. 2

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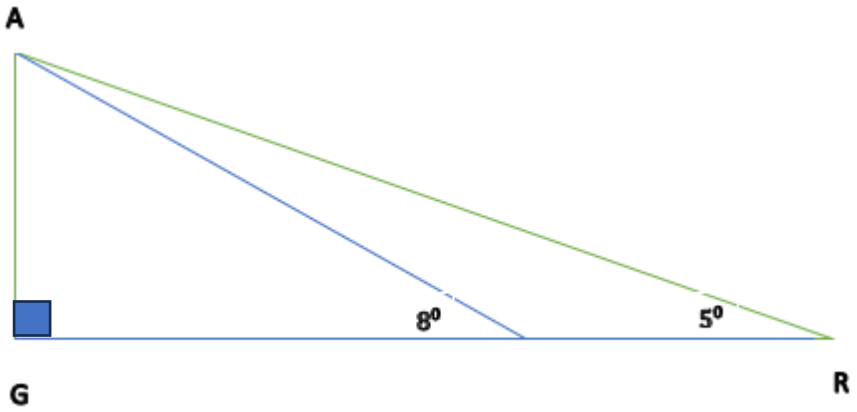
Proceed to Answer Booklet 3 for Questions 26-31

Question 27 (4 marks)



Roberto, ***R*** in the World cup, knows that from his position directly in front of the goalkeeper, ***G***, he must kick the ball through the point ***A*** to score a goal. If he runs the ball forward an extra five metres, his kicking angle increase from 5° to 8°.

a. Calculate Roberto's kicking distance to ***A,*** from the 8° mark, correct to 2 decimal places. 2

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b. Find how far Roberto was from the goalkeeper at the 5° mark, correct to 1 decimal place. 2

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Question 28 (5 marks)

A farmer accidently spread a dangerous chemical on a paddock. The concentration of the chemical in the soil was initially measured at 5 kg/ha. One year later the concentration was found to be half of the initial concentration.

It is known that the concentration, C, is given by $C = C_0 e^{-kt}$, where C_0 and k are constants, and t is measured in years.

a. Evaluate C_0 and k. Leave your answers in exact forms. 3

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b. It is safe to use the paddock when the concentration is below 0.2 kg/ha. How long must the farmer wait after the accident before the paddock can be used? Give your answer in years, correct to one decimal place. 2

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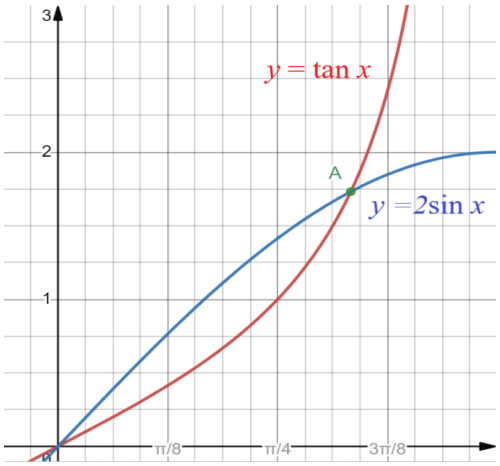
Question 29 (4 marks)



The diagram above shows the curves $y = \tan x$ and $y = 2\sin x$ for $0 \leq x \leq \frac{\pi}{2}$.

a. Show that the coordinates of A are $\left(\frac{\pi}{3}, \sqrt{3}\right)$. 1

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b. Show that $\frac{d}{dx}(\ln \cos x) = -\tan x$. 1

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c. Hence, find the area between $y = \tan x$ and $y = 2\sin x$ for $0 \leq x \leq \frac{\pi}{2}$. 2

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Question 30 (4 marks)

a. Complete the table below for $y = \sqrt{\cos x}$. 2

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y			0.841		

b. Hence estimate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\ dx$ using the trapezoidal rule with 4 applications. 2

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Question 31 (8 marks)

Mr Pollard borrowed $500 000 from his bank to buy an apartment. The loan is to be repaid in equal monthly instalments. The interest rate is 8.4% p.a., calculated monthly.

a. Show that the interest for the first month is $3500. 1

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b. Why is it not advisable that Mr Pollard's monthly repayments be $3500? 1

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c. Mrs Briggs thinks Mr Pollard should cut down on his unnecessary expenses and budget himself to pay $5000 per month off the loan.

Show that $A_2 = 500000 \times 1.007^2 - 5000(1 + 1.007)$. where A_2 is the amount owing after two repayments, 3

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d. Hence, find an expression for A_n, the amount owing after the nth repayment. 1

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e. How long will it take for Mr Pollard to pay off the loan? 2

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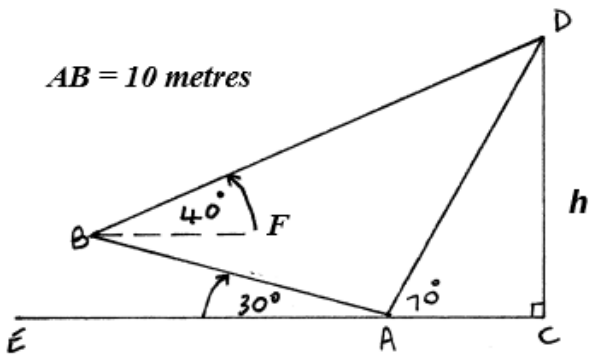
Question 32 (5 marks)

Lina is standing on a footpath at point A. From this position she looks up to the top of a vertical tower, CD, at an angle of elevation of 70°. Lina then turns around and walks up 10 m up a ramp AB, inclined at an angle of 30° to the horizontal footpath. From her new position, B, the top of the tower is observed at an angle of elevation of 40°. This information is shown in the diagram below.



a. Find the size of $\angle ABD$, and of $\angle BDA$. **DO NOT** provide any reasons. 2

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b. Find the length of AD. 2

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c. Hence, find the height of the tower correct to 1 decimal place. 1

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END OF EXAMINATION

2023

Penrith High School

HIGHER SCHOOL CERTIFICATE

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							Total:	/10

Differentiation And applications of Differential Calculus	Statistics	Algebra & Graphing Functions	Integration	Logarithms & Exponentials	Probability	Series & Financial mathematics	Trigonometric functions
/21	/5	/6	/16	/6	/11	/13	/12
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Student NESA Number _____

Teacher_____

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   A. 128

   B. -128

   C. 0

   D. 64

2. A snail starts moving in a garden and covers 6 metres in the first hour. If the snail covers only half the distance covered in the previous hour in each successive hour, how far will the snail be able to travel?

   A. 10.5 metres

   B. 11.25 metres

   C. 11.625 metres

   D. 12 metres

3. Which of the following expressions is equivalent to $4 + \ln x$?

   A. $\ln(4x)$

   B. $\ln(4e + x)$

   C. $\ln(e^4 x)$

   D. $\mathrm{e}^4 \ln(x)$

4. The discrete random variable X has the following distribution:

X	0	1	2	3
$P(X = x)$	0.15	a	0.35	$2b$

Given that $\mathrm{E}(\mathrm{X}) = 1.5$, find the values of a and b.

A. $a = 0.15$ and $b = 0.7$

B. $a = 0.35$ and $b = 0.075$

C. $a = 0.35$ and $b = 0.15$

D. $a = 0.15$ and $b = 0.35$

5. \$1350 is invested into an account earning 3% p.a. for 5 years. Interest is compounded quarterly. Which expression correctly evaluates the amount of interest earned?

A. 1350×1.0075^{20}

B. 1350×1.03^{5}

C. $1350 \times 1.0075^{20} - 1350$

D. $1350 \times 1.03^{5} - 1350$

6. A dice is rolled twice and the uppermost numbers are added together.

Which of the following events has a probability of $\frac{1}{18}$?

A. Both numbers rolled are the same

B. The sum is 5

C. The sum is 3

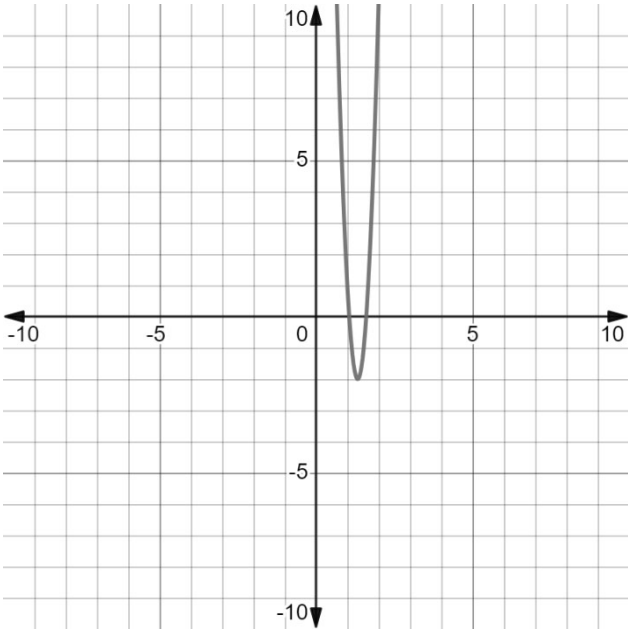
D. The sum is 12

7. The graph of $y = x^2$ undergoes the following sequence of transformations:

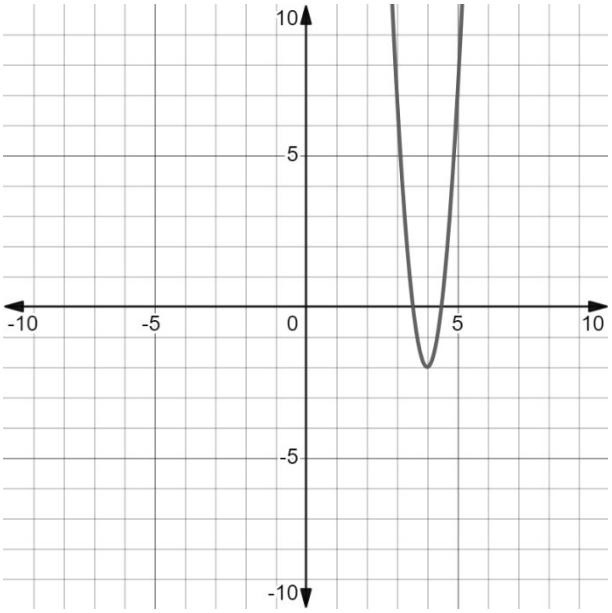
- *Translation 4 units right*
- *Dilation horizontally by a factor of* $\frac{1}{3}$
- *Translation 2 units down*
- *Dilation vertically by a factor of 3*

The graph of the resulting equation is given by

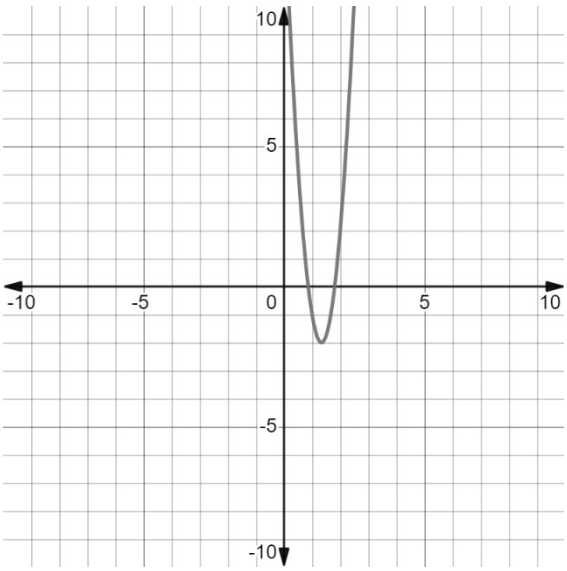
A.



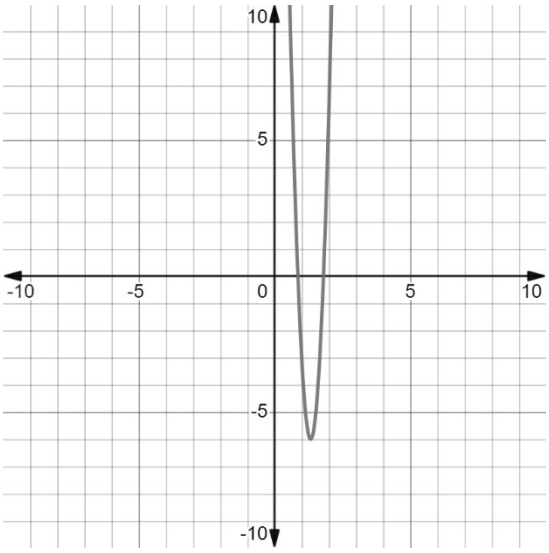
B.



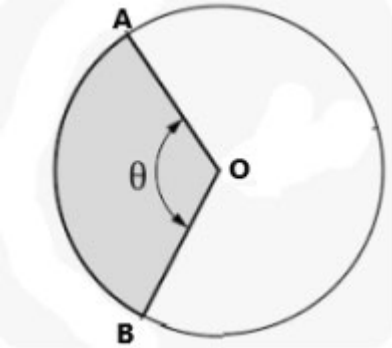
C.



D.



8. In a circle, an arc AB subtends an angle of θ radians at the centre shown below. The radius of the circle is $r\ cm$ and the perimeter of the minor sector AOB is $4\ cm$.



The angle θ and the area of the sector AOB is given by

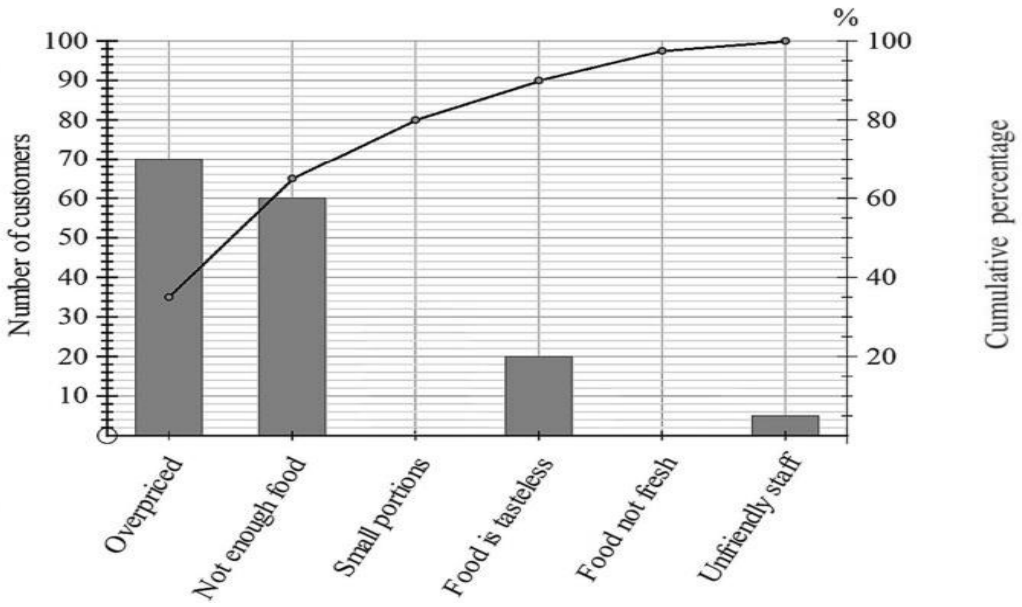
A. $\theta = \frac{4}{r} - 2$ and $A = 2r - r^2$

B. $\theta = \frac{4}{r}$ and $A = 2r$

C. $\theta = \frac{4}{r} - 2$ and $A = 4r - r^2$

D. $\theta = \frac{4}{r}$ and $A = 4r$

9.



The Pareto chart given above shows the complaints of 200 customers from a restaurant in the area. The columns for the "small portions" and "food not fresh" complaints are missing.

The approximate total number of complaints for "Small portions" is

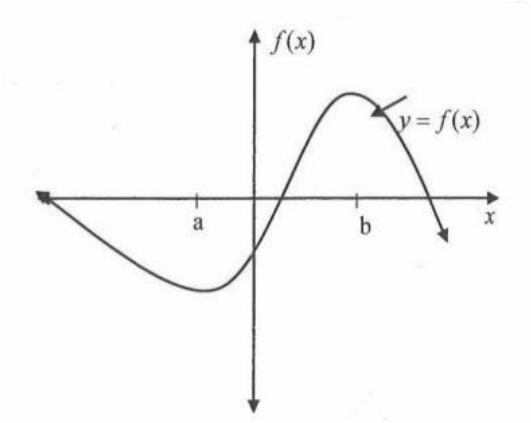
A. 45 people

B. 30 people

C. 40 people

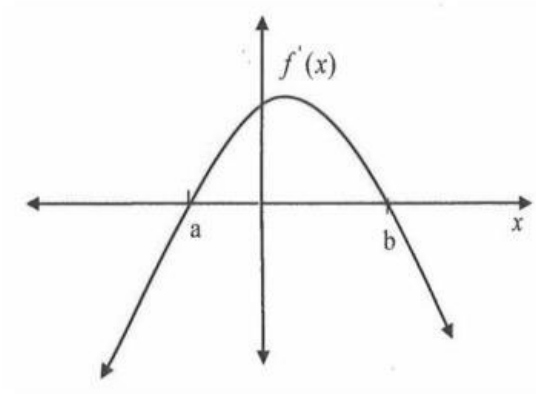
D. 80 people

10. The diagram given below shows the graph of $y = f(x)$.

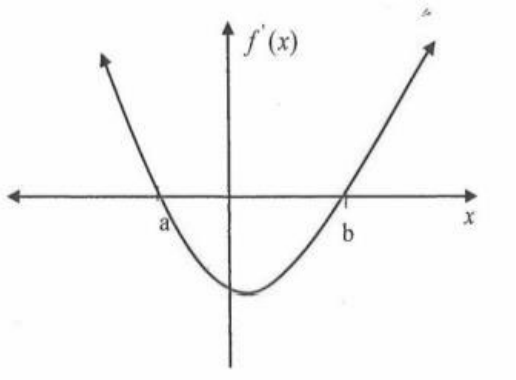


Which of the following shows the graph of $y = f'(x)$?

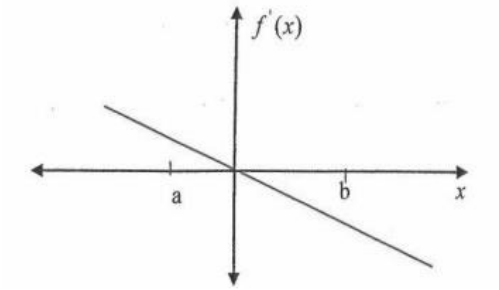
A.



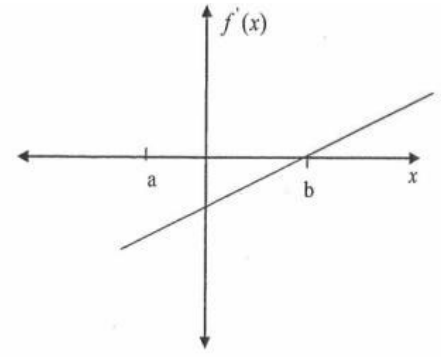
B.



C.



D,



End of SECTION I

NESA Number: | | | | | | | | | |

2023

Penrith High School
HSC TRIAL
EXAMINATION

Teacher: _____ ____

Mathematics Advanced

Section II Answer Booklet 1

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section.

Booklet 1 – Attempt Questions 11 – 20 (30 marks)
Booklet 2 – Attempt Questions 21 – 26 (30 marks)
Booklet 3 – Attempt Questions 27 – 32 (30 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above.

Trigonometric Functions	Algebra & Graphing	Integration	Series & Financial Mathematics	Probability	Differentiation & Applications of Differentiation	Logarithms & Exponentials
/3 13	/6 14 17	/3 15	/3 16	/9 12 18	/3 11	/3 19

Please Turn Over

$f(x+h) = 2(x+h)^2 + (x+h)$

Question 11 (3 marks)

A student was asked to differentiate $f(x) = 2x^2 + x$ using the first Principles of Differential Calculus. The student began the solution as shown below.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Complete the solution. 3

* Students need to take more care setting out their work!

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h}$$

Some students forget these terms

$$= \lim_{h\to 0} \frac{2(x^2 + 2xh + h^2) + x + h - 2x^2 - x}{h}$$

Common mistake expanding incorrectly

$$= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h}$$ (1)

$$= \lim_{h\to 0} \frac{4xh + 2h^2 + h}{h}$$

$$= \lim_{h\to 0} \frac{h(4x + 2h + 1)}{h}$$ (1)

$$= \lim_{h\to 0} 4x + 2(0) + 1$$

Common mistakes with factorising

$$= 4x + 1$$ (1)

Overall, done well.

Question 12 (1 mark)

A and B are events of a sample space. Given that $P(A \cap B) = 0.15$ and $P(A|B) = 0.2$, calculate $P(B)$ for this sample space. 1

$P(A \cap B) = 0.15$

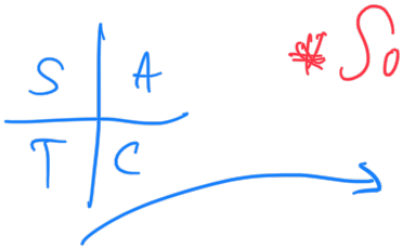
$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$0.2 = \frac{0.15}{P(B)}$

$0.2\,P(B) = 0.15$

$P(B) = \frac{0.15}{0.2} = 0.75$ (1)

* This question was done well



* Some students were missing angles $0 \le 2\theta \le 4\pi$

Question 13 (3 marks)

Solve $2\sin 2\theta - 1 = 0$ for $0 \le \theta \le 2\pi$. 3

$\sin 2\theta = \frac{1}{2}$ Some students made a mistake with the first step.

$2\theta = \frac{\pi}{6}$ (1), $\pi - \frac{\pi}{6}$, $2\pi + \frac{\pi}{6}$, $3\pi - \frac{\pi}{6}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ (1)

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{6}$ (1)

* Some students multiplied by 2 instead of divide by 2.

Question 14 (3 marks)

Let $f(x) = x - 2$ and $g(x) = \frac{1}{x} + 1$,

a. find $g(f(x))$. 1

$g(f(x)) = \frac{1}{x-2} + 1$

* Done well

b. Find the domain and range of $g(f(x))$ in interval notation. 2

Domain: (1) $(-\infty, 2) \cup (2, \infty)$ * Students used the wrong symbols.

Range: (1) $(-\infty, 1) \cup (1, \infty)$

* Not done well

Question 15 (3 marks)

A function $f(x)$, has a derivative given by $f'(x) = 3x^2 - x + 6$. The function curve passes through the point $(0, 1)$. Find the equation of the function. 3

$f(x) = \int 3x^2 - x + 6 \, dx$

$= \frac{3x^3}{3} - \frac{x^2}{2} + 6x + C$ (1)

$f(x) = x^3 - \frac{x^2}{2} + 6x$

substitute $(0, 1)$:

$1 = 0^3 - \frac{0^2}{2} + 6(0) + C$ } Some students substituted the wrong values.

$\therefore C = 1$ (1)

$\therefore f(x) = x^3 - \frac{x^2}{2} + 6x + 1$ (1)

* Done well overall.

$T_n = a + (n-1)d$

* Some students wrote the formula wrong.

Question 16 (3 marks)

The second term of an arithmetic sequence is 12. The fifteenth term is 64.

Calculate the 25th term. 3

* This question was done well

$T_2 = a + d$

$12 = a + d \longrightarrow d = 12 - a$

$\therefore d = 4$ (1)

$T_{15} = a + 14d$

$64 = a + 14(12 - a)$ (1)

$64 = a + 168 - 14a$

$-128 = -13a$

$a = 8$

$T_{25} = 8 + (25-1)4$

$= 8 + 24(4)$

$= 104$ (1)

$\therefore$ 25th term is 104.

Question 17 (3 marks)

Consider the curve given by $f(x) = x^2 \ln|x|$ 3

Is $f(x) = x^2 \ln|x|$ an odd or an even function or neither? Justify your response.

$f(x) = x^2 \ln|x|$

$f(-x) = (-x)^2 \ln|-x|$

$= x^2 \ln|x|$ (1)

$= f(x)$

Some students left out a description when they removed the absolute value symbol losing 1 mark.

Not done well

$\therefore f(x)$ is an even function (1)

since $f(-x) = f(x)$ (1)

Question 18 (8 marks)

The probability distribution for a discrete random variable X is given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3 \\ k(6 - x) & x = 4, 5, 6 \end{cases}$$

probabilities add to 1

a. Show that $k = \frac{1}{9}$. 1

substitute the x-values:

Not done well

$k + 2k + 3k + k(6-4) + k(6-5) + k(6-6) = 1$ (1)

$k + 2k + 3k + 2k + k + 0 = 1$

$9k = 1$

$k = \frac{1}{9}$

b. Complete the Probability distribution table given below. 2

Done well.

X	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	0

(1) (1)

c. Hence, calculate its expected value. 2

$E(X) = 1 \times \frac{1}{9} + 2 \times \frac{2}{9} + 3 \times \frac{3}{9} + 4 \times \frac{2}{9} + 1 \times \frac{1}{9} + 6 \times 0$ (1)

Done well.

$= \frac{1}{9} + \frac{4}{9} + \frac{9}{9} + \frac{8}{9} + \frac{1}{9} + 0$

$E(X) = \frac{27}{9} = 3 \quad \therefore \mu = 3$ (1)

d. Calculate the variance and the standard deviation correct to 2 decimal places. 2

$Var(X) = \sum (x^2)p(x) - \mu^2$

$= 1 \times \frac{1}{9} + 4 \times \frac{2}{9} + 9 \times \frac{3}{9} + 16 \times \frac{2}{9} + 25 \times \frac{1}{9} + 0$

$= \left(\frac{1}{9} + \frac{8}{9} + 3 + \frac{32}{9} + \frac{25}{9}\right) - 3^2$

$Var(X) = \frac{4}{3}$ (1) $\quad \sigma = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = 1.15$ (to 2dp) (1)

$= 1.33$ (to 2dp)

Students did not give an answer for standard deviation.

e. Find $P(2 \leq X \leq 4)$. 1

$\frac{2}{9} + \frac{3}{9} + \frac{2}{9} = \frac{7}{9}$ (1)

* Overall, this question most students correctly answered this.

* Students who did not use logarithmic laws made the question harder than it should be.

Question 19 (3 marks)

Using the logarithm laws, show that $\frac{d}{dx}\left(\ln\sqrt{\frac{1+x}{1-x}}\right)=\frac{1}{1-x^2}$ 3

LHS $= \frac{d}{dx}\ln\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} = \frac{1}{2}\left[\ln(1+x)-\ln(1-x)\right]$ (1)

$= \frac{1}{2}\left(\frac{1}{1+x}-\frac{1}{1-x}\times(-1)\right)$

$= \frac{1}{2}\left[\frac{1}{1+x}+\frac{1}{1-x}\right]$

$= \frac{1}{2}\left[\frac{1-x+1+x}{(1+x)(1-x)}\right]$ (1)

$= \frac{1}{2}\left(\frac{2}{(1+x)(1-x)}\right)$ (1)

$= \frac{1}{1-x^2}$

$=$ RHS

Method 2: Some students used the quotient rule:

but $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

* Students did not divide by $\sqrt{\frac{1+x}{1-x}}$ hence lost a mark.

Proceed to Answer Booklet 2 for Questions 20-24

Mathematics Advanced

Student Number .....

Solutions with feedback

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 20 – 26 (30 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above.

Differentiation	Integration	Probability	Series & Financial Mathematics	Statistics	Logarithms & Exponential
/13 20 22a 26	/5 24	/2 25	/2 21	/5 23	/3 22b

Please Turn Over

Question 20 (8 marks)

Consider the curve given by $f(x) = 2x^3 - 5x^2 + 6$.

a. Find the coordinates of the stationary points and determine for their nature. 3

$f'(x) = 6x^2 - 10x$

$f''(x) = 12x - 10$ (1)

stat. points when $f'(x) = 0$

$\therefore 6x^2 - 10x = 0$

$2x(3x - 5) = 0$

$\therefore x = 0$ or $x = 5/3$

when $x = 0$, $y = 6$ $\therefore (0, 6)$

when $x = 5/3$, $y = 37/27$ $\therefore (5/3, 37/27)$ (1)

when $x = 0$, $f''(0) = -10 < 0$ $\therefore$ concave down

$\therefore$ maximum at $(0, 6)$

when $x = 5/3$, $f''(5/3) = 10 > 0$ concave up

$\therefore$ minimum at $(5/3, 37/27)$

(1) for both derivatives correct ← (1) →

(1) for stat. pts as coordinates, ie (x_1, y_1) & (x_2, y_2)

(1) for testing the nature of both stat. pts.

b. Find the coordinates of any point(s) of inflection. 2

possible points of inflexion when $f''(x) = 0$

$\therefore 12x - 10 = 0$

$x = 5/6$

when $x = 5/6$ $f(5/6) = 2(5/6)^3 - 5(5/6)^2 + 6 = \frac{199}{54}$ (1)

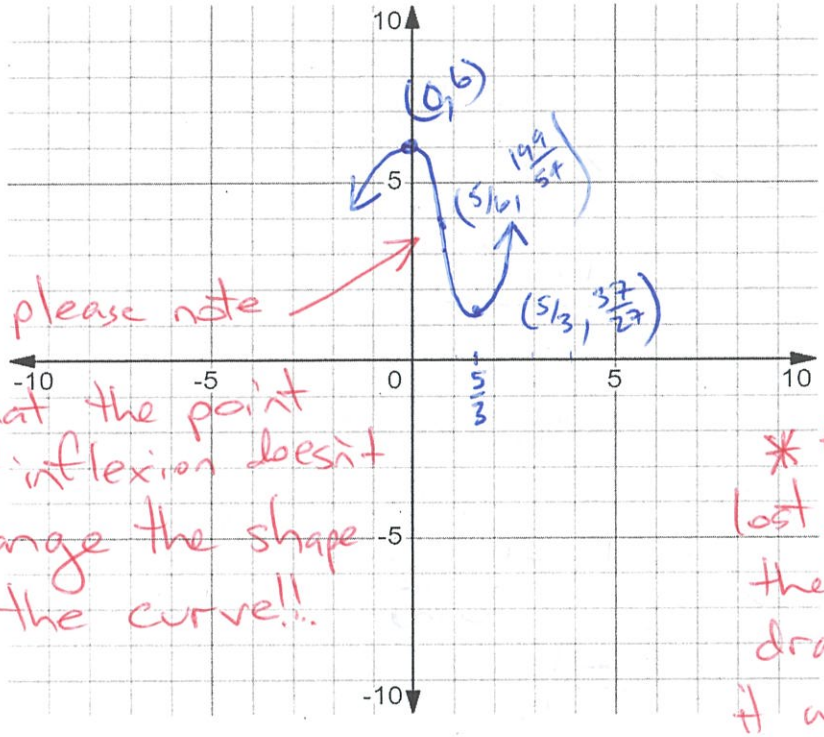
x	$\frac{1}{2}$	$5/6$	1
$f''(x)$	-4	0	2
concavity	down	—	up

$\therefore$ change in concavity

$\therefore$ pt. of inflexion at $(5/6, \frac{199}{54})$ (1)

*A lot of students forgot to test, and show there was a change in concavity!!!

c. Hence, sketch the curve labelling the stationary points, point(s) of inflection (if any) and $y-$ intercept. Do Not determine the $x-$ intercepts of the curve. 3



please note that the point of inflexion doesn't change the shape of the curve!!..

1 for shape
1 for y-intercept
1 for labelling.

* A lot of students lost a mark because their curve wasn't drawn neatly, or it was on an angle.

eg

Question 21 (2 marks)

Consider the series given by $1 + 3x + 9x^2 + 27x^3 + \cdots \ldots$

a. Show that it is a geometric series. 1

If $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$ then its a G.P.

$\frac{T_2}{T_1} = \frac{3x}{1} = 3x$ $\quad \frac{T_3}{T_2} = \frac{9x^2}{3x} = 3x$ $\quad \frac{T_4}{T_3} = \frac{27x^3}{9x^2} = 3x$

$\therefore$ its a G.P. with $r = 3x$

* not done well.
* A lot of students used the geometric formula

b. Find the range of values of x for which the series has a limiting sum. 1

$|3x| < 1$

$|x| < \frac{1}{3}$ ← a lot of students couldn't solve this inequality

$\therefore -\frac{1}{3} < x < \frac{1}{3}$ or $\left(-\frac{1}{3}, \frac{1}{3}\right)$

Question 22 (6 marks)

a. Using the quotient rule, show that $\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = -cosec^2 x$ 3

$u = \cos x$, $v = \sin x$ $\qquad \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u' = -\sin x$, $v' = +\cos x$

$d/dx = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$ (1)

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

(1) $= -1\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x}$ ← a lot of students omitted this step! ∴

$= \frac{-1}{\sin^2 x}$

(1) $= -\operatorname{cosec}^2 x$ } need to see both lines to get the final mark

b. Evaluate $\int_5^8 \frac{x-2}{x^2-4x}\,dx$. Leave your answer in exact form. 3

$= \frac{1}{2}\int_5^8 \frac{2x-4}{x^2-4x}\,dx$ $\qquad \int \frac{f'(x)}{f(x)}\,dx$

$= \frac{1}{2}\left[\ln(x^2-4x)\right]_5^8$ (1) correct integration

$= \frac{1}{2}\left[\ln(64-32) - \ln(25-20)\right]$ (1) correct substitution

$= \frac{1}{2}(\ln 32 - \ln 5)$

$= \frac{1}{2}\ln \frac{32}{5}$ (1) application of log laws.

Question 23 (5 marks)

Six Penrith High School Year 12 Students participated in the javelin and discus throwing events at the Athletics Carnival this year. Their distances (in metres) were recorded in the table below.

Student Names	Distance (Javelin)	Distance (Discus)
Saatvik	8	18
Rithwik	60	81
Yesh	51	57
Angelo	48	48
Darwin	29	25
Lisa	36	47

The relationship between the two events is being examined. The distance thrown in javelin is the independent variable and the distance thrown in discus is the dependent variable.

a. Calculte the Pearson correlation coefficient, r, for the above data set correct to 3 decimal places. Explain clearly what this value indicates in this context. (2)

(1) Pearson correlation coefficient is $r = 0.927$

(1) strong postive and high correlation, indicates a strong link between the 2 skills.

b. Calculate the equation of the least squres regression line, rounding numbers correct to 1 decimal place where necessary. (2)

$y = 2.3 + 1.1x$ (1) (1)

* Some students just wrote "$2.3 + 1.1x$", this is an expression not an equation. Be more careful.

c. Saatvik concluded after the event that "Yesh was able to throw his javelin a greater distance because he threw his discus a greater distance, and being successful in one skill transferred to the other skill". Saatvik's statement may not be correct. Why? 1

Although there's a strong postive relationship between the 2 skills, indicated by $r=0.927$, the ability to throw a javelin far doesn't mean in reality that you can throw a discus far, It's 2 different types of "throws"

Question 24 (5 marks)

The acceleration of a particle at any time t is given by $\ddot{x} = 2\cos\frac{\pi t}{3}$.

a. Find the velocity and displacement of the particle at any time t, given that the particle is initially at rest at the origin. 3

$\ddot{x} = 2\cos\frac{\pi t}{3}$

$\dot{x} = \frac{2\sin\frac{\pi}{3}t}{\frac{\pi}{3}} + c$

a lot of students forgot this

$\dot{x} = \frac{6}{\pi}\sin\frac{\pi}{3}t + c$ (1)

when $t=0$, $\dot{x}=0$ $\therefore c=0$

$\therefore \dot{x} = \frac{6}{\pi}\sin\frac{\pi}{3}t$

$x = \frac{-\frac{6}{\pi}\cos\frac{\pi}{3}t}{\frac{\pi}{3}} + K$ (1)

$x = \frac{-18}{\pi^2}\cos\frac{\pi}{3}t + K$

when $t=0$, $x=0$

$\therefore 0 = \frac{-18}{\pi^2}\cos(0) + K$

$\therefore 0 = \frac{-18}{\pi^2} + K$

$K = \frac{18}{\pi^2}$

a lot of students assumed that $K=0$.

$\therefore x = \frac{-18}{\pi^2}\cos\frac{\pi}{3}t + \frac{18}{\pi^2}$ (1)

*you need different constant of integrations, as they have 2 different values,

b. Find the time when the particle first returns to the origin. 2

when $x=0$, $t=??$

$\therefore 0 = \frac{-18}{\pi^2}\cos\frac{\pi}{3}t + \frac{18}{\pi^2}$

$\frac{18}{\pi^2}\cos\frac{\pi}{3}t = \frac{18}{\pi^2}$

$\cos\frac{\pi}{3}t = 1$ (1)

$\frac{\pi}{3}t = 0, 2\pi, 4\pi, \ldots$

$\pi t = 0, 6\pi, 12\pi \ldots$

$t = 0, 6, 12, \ldots$

$\therefore$ The particle first returns to the origin at $t = 6$ seconds (1)

*If students had a wrong, they got "carried forward" error here

Question 25 (2 marks)

A survey of a certain district showed that 4% of the families have 1 child, 34% have 2 children, 40% have 3 children, and 15% have more than 3 children. A family from the district is selected at random. Find the probability that the family will have at most 1 child. 2

$100\% - 4\% - 34\% - 40\% - 15\% = 7\%$ (1)

$P(\text{at most 1 child}) = 4\% + 7\% = 11\%$ or $\frac{11}{100}$ (0.11) (1)

Question 26 (2 marks)

Show that the graph given by $f(x) = \frac{1}{1+e^{-x}}$, $x \geq 0$ is an increasing function for all values of x in the given domain. 2

$f(x) = (1+e^{-x})^{-1}$

$f'(x) = -1(1+e^{-x})^{-2} \times -e^{-x}$

(1) $= e^{-x}(1+e^{-x})^{-2}$

$= \frac{e^{-x}}{(1+e^{-x})^2}$

now $e^{-x} > 0$ for all x.

$(1+e^{-x})^2 > 0$ for all x.

since $f'(x) > 0$ for all x, the function is increasing. (1)

needed all of this to get the 2nd mark.

Proceed to Answer Booklet 3 for Questions 26-31

* this question was badly done.
* very chaotic in the way students set out their answers - think, plan and then write logically!

Mathematics Advanced

Student Number ……

Section II Answer Booklet 3

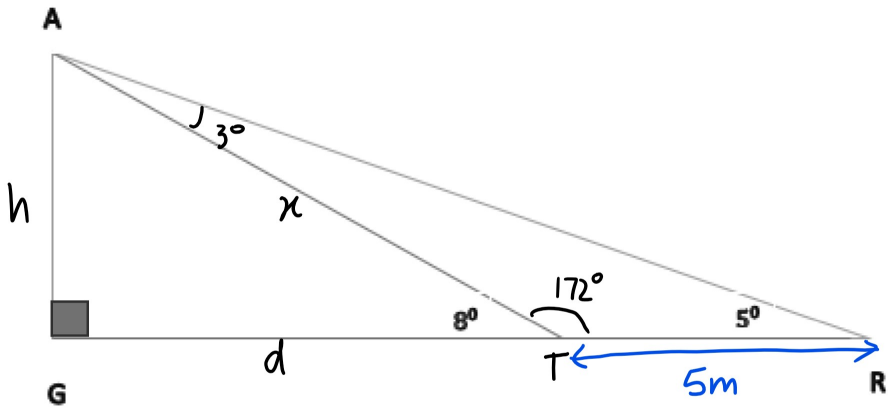
Booklet 3 – Attempt Questions 27 – 32 (30 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above.

Differentiation	Trigonometry	Integration	Series & Financial Mathematics
/5 28	/9 27 32	/8 29 30	/8 31

Question 27 (4 marks)



Roberto, ***R*** in the World cup, knows that from his position directly in front of the goalkeeper, ***G***, he must kick the ball through the point ***A*** to score a goal. If he runs the ball forward an extra five metres, his kicking angle increase from 5° to 8°.

a. Calculate Roberto's kicking distance to ***A***, from the 8° mark, correct to 2 decimal places. 2

Sine rule: $\frac{x}{\sin 5} = \frac{5}{\sin 3}$

$x = \frac{5 \sin 5}{\sin 3}$ ✓

$= 8.33\text{m}$ ✓

Common mistakes:
- students calculated horizontal distance instead of hypotenuse.
- used simultaneous equations with two right angled triangles.

b. Find how far Roberto was from the goalkeeper at the 5° mark, correct to 1 decimal place. 2

$\cos 8 = \frac{d}{8.32}$

$d = 8.32 \times \cos 8$ ✓

$= 8.245$

$d + 5 = 8.245 + 5$ ✓

$= 13.2\text{ m}$ away at 5° mark.

Mostly well done.

Question 28 (5 marks)

A farmer accidently spread a dangerous chemical on a paddock. The concentration of the chemical in the soil was initially measured at 5 kg/ha. One year later the concentration was found to be half of the initial concentration.

It is known that the concentration, C, is given by $C = C_0 e^{-kt}$, where C_0 and k are constants, and t is measured in years.

a. Evaluate C_0 and k. Leave your answers in exact forms. 3

When $t=0$, $5 = C_0(1)$ ✓ 1 mark for C_0

$2.5 = 5e^{-k}$ when $t=1$

$0.5 = e^{-k}$

$-k = \ln 0.5$ ✓ 2 marks for k, both forms accepted.

$k = -\ln 0.5 \longrightarrow k = \ln 2$

b. It is safe to use the paddock when the concentration is below 0.2 kg/ha. How long must the farmer wait after the accident before the paddock can be used? Give your answer in years, correct to one decimal place. 2

$0.2 = 5e^{\ln 0.5t}$

$e^{\ln 0.5t} = \frac{0.2}{5}$

$e^{\ln 0.5t} = 0.04$

Majority of students wrote $t = 4.7$ years.

$\ln e^{\ln 0.5t} = \ln 0.04$ ✓

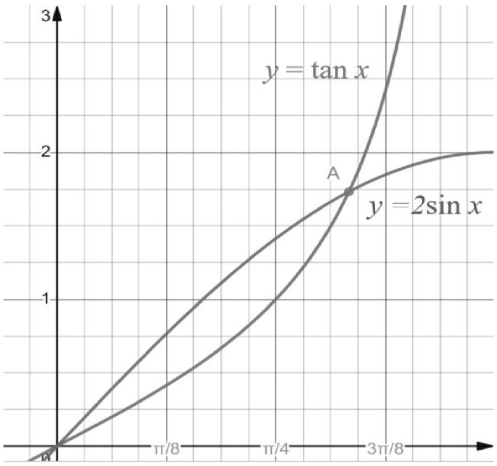
$\ln 0.5t = \ln 0.04$

$t = \frac{\ln 0.04}{\ln 0.5}$

$t = 4.64$ years ✓

Rounding to 1 decimal place, $t = 4.6$ years.
However this is not safe and rounding to $t = 4.7$ years is more appropriate.

Question 29 (4 marks)



The diagram above shows the curves $y = \tan x$ and $y = 2\sin x$ for $0 \leq x \leq \frac{\pi}{2}$.

a. Show that the coordinates of A are $\left(\frac{\pi}{3}, \sqrt{3}\right)$. 1

sub $(\pi/3, \sqrt{3})$ into $y = \tan x$. LHS $= \sqrt{3}$, RHS $= \tan(\pi/3) = \sqrt{3}$ = LHS

sub $(\pi/3, \sqrt{3})$ into $y = 2\sin x$, LHS $= \sqrt{3}$, RHS $= 2\sin\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2}$ = LHS ✓

b. Show that $\frac{d}{dx}(\ln\cos x) = -\tan x$. 1

$\frac{-\sin x}{\cos x} = -\tan x$ ✓

c. Hence, find the area between $y = \tan x$ and $y = 2\sin x$ for $0 \leq x \leq \frac{\pi}{2}$. 2

From part (a)

$\int_0^{\pi/3} 2\sin x - \tan x \, dx$

$= -2[\cos x]_0^{\pi/3} + [\ln\cos x]_0^{\pi/3}$ using part (b) ✓

$= -2[\cos(\pi/3) - \cos 0] + [\ln\cos\pi/3 - \ln\cos 0]$

$= -2\left[\frac{1}{2} - 1\right] + \left[\ln\left(\frac{1}{2}\right)\right]$ ✓

$= 1 - \ln 2$

Common mistake:
- $2\sin x$ is in front of $\tan x$.
- students converted $(-\tan x)$ into $(-\ln\cos x)$

Question 30 (4 marks)

a. Complete the table below for $y = \sqrt{\cos x}$. 2

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	1	0.961	0.841	0.691	4

1 mark for each pair.

b. Hence estimate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\ dx$ using the trapezoidal rule with 4 applications. 2

$$\int_0^{\pi/2} \sqrt{\cos x}\ dx = \frac{\pi/2 - 0}{2(4)}\left[(1+0)+2(0.961+0.841+0.691)\right]$$

$$= \frac{\pi}{16}\left[(1)+2(2.421)\right]$$

$$= \frac{\pi}{16}(5.842)\ u^2$$

$$= 1.1471\ u^2$$

Some students did got the incorrect. $\frac{b-a}{2n}$

Question 31 (8 marks)

Mr Pollard borrowed $500 000 from his bank to buy an apartment. The loan is to be repaid in equal monthly instalments. The interest rate is 8.4% p.a., calculated monthly.

a. Show that the interest for the first month is $3500. 1

$$500\,000\,(1.007) = \$503\,500$$

$$I = A - P$$

$$= 503500 - 50000$$

$$= \$3500$$

Very well done.

b. Why is it not advisable that Mr Pollard's monthly repayments be \$3500? 1

This means he will not pay off and of the loan amount, and amount owing on the loan remains the same.

Students answers were vague and did not address the application context.

c. Mrs Briggs thinks Mr Pollard should cut down on his unnecessary expenses and budget himself to pay \$5000 per month off the loan.

Show that $A_2 = 500000 \times 1.007^2 - 5000(1 + 1.007)$. where A_2 is the amount owing after two repayments, 3

$A_1 = 500000\,(1.007) - 5000$

$A_2 = A_1\,(1.007) - 5000$

$= 500000\,(1.007)^2 - 5000\,(1.007) - 5000$

Mostly well done.

Eaglest 3 marker.

$A_2 = A_1 \times (1.007) - 5000$

$= 500000\,(1.007)^2 - 5000\,(1.007) - 5000$

$= 500000\,(1.007)^2 - 5000\,(1 + 1.007)$

d. Hence, find an expression for A_n, the amount owing after the nth repayment. 1

$A_n = 500000\,(1.007)^n - 5000\,(1 + \ldots + 1.007^{n-1})$

Mostly well done.

Students tried to simplify when not needed to.

e. How long will it take for Mr Pollard to pay off the loan? 2

$A_n = 0$ GP

$0 = 500\,000(1.007)^n - 5000\left[1 + \ldots + 1.007^{n-1}\right]$

$0 = 500000(1.007)^n - 5000\left[\frac{1(1.007^n - 1)}{1.007 - 1}\right]$

$\frac{5000}{0.007} \times (1.007^n - 1) = 500\,000(1.007)^n$ ✓ set up

$714\,285.7143(1.007)^n - 500000(1.007)^n = 714285.7143$

$(1.007)^n = 3\frac{1}{3}$

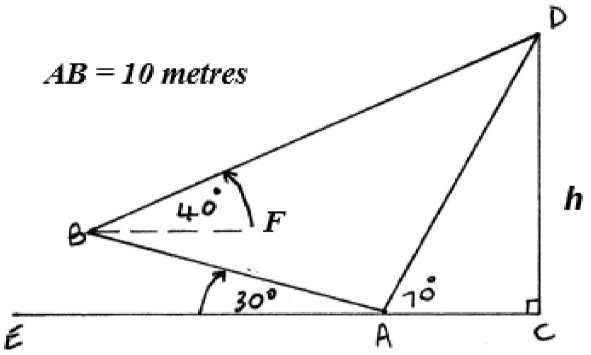
$n \ln(1.007) = \ln(3\frac{1}{3})$

$n = 172.597$ ✓ final answer

$= 173$ months

Question 32 (5 marks)

Lina is standing on a footpath at point A. From this position she looks up to the top of a vertical tower, CD, at an angle of elevation of 70°. Lina then turns around and walks up 10 m up a ramp AB, inclined at an angle of 30° to the horizontal footpath. From her new position, B, the top of the tower is observed at an angle of elevation of 40°. This information is shown in the diagram below.



a. Find the size of $\angle ABD$, and of $\angle BDA$. **DO NOT** provide any reasons. 2

$\angle ABD = 70°$ ✓ Well done.

$\angle BDA = 180 - 80 - 70$

$= 30°$ ✓

b. Find the length of AD. 2

Sine rule: $\frac{AD}{\sin 70} = \frac{10}{\sin 30}$

$AD = \frac{10 \times \sin 70}{\sin 30}$ ✓ Well done

$\therefore AD = 18.79\text{ m}$ ✓

c. Hence, find the height of the tower correct to 1 decimal place. 1

$\sin 70 = \frac{h}{18.79}$

$h = 18.79 \sin 70$ ✓ CFE if needed.

$= 17.66\text{m}$

$= 17.7\text{m}$

END OF EXAMINATION